Opt. Commun. 283, 1285(2010)
Corresponding author A. Lencina agl@ciop.unlp.edu.ar

Cluster speckle structures through multiple apertures forming a closed curve

E. Mosso [a], M. Tebaldi [a,b,*], A. Lencina [a], N. Bolognini [a,c]

[a] Centro de Investigaciones Ópticas (CONICET CCT LA PLATA-CIC), P.O. Box 3, 1897 La Plata, Argentina
[b] UID OPTIMO, Facultad de Ingeniería, Universidad Nacional de La Plata, La Plata, Argentina
[c] Facultad de Ciencias Exactas Universidad Nacional de La Plata, La Plata, Argentina

In this work, cluster-like speckle patterns are analyzed. These patterns are generated when a diffuser illuminated by coherent light is imaged by a lens having a pupil mask with multiple apertures forming a closed curve. We show that the cluster structure results from the complex modulation produced inside each speckle which is generated by multiple interferences of light through the apertures. In particular, when the apertures are uniformly distributed along a closed curve, the resulting image speckle cluster replicates the pupil aperture distribution. Experimental results and theoretical simulations show that cluster features depend on the apertures distribution and the size of the closed curves.

1. Introduction

Speckle patterns are random intensity distributions formed when coherent light either reflects from a rough surface or propagates through a medium which has random optical path fluctuations. In general, speckle statistical properties depend both on the coherence of the incident light and the properties of the random surface. When considering a perfect coherent light and scatters which introduce path differences greater than one wavelength, then the speckle dependence on the random scatters is almost negligible. Besides, the lateral structure of a random distribution is strictly defined in terms of the autocorrelation function [1,2]. In fact, the minimum speckle size approximates to the Airy disk size that would be produced in the absence of the random medium.

The use of speckle patterns in the study of object displacements that arises in nondestructive testing of mechanical components is an important field of analysis [1,2]. The key advantage of speckle methods relay on that the speckle size may be adjusted to suite the resolution of the most convenient detectors. Also, this behavior still maintains the information about displacements on an interferometric scale if required. To a certain extent, metrological speckle techniques have been implemented on the basis of the speckle internal modulation which can be achieved by attaching a pupil mask with several apertures behind the imaging lens [3–5]. The mentioned speckle modulation not only exhibits more complexity but it offers larger capabilities in terms of potential applications [6–8].

It should be pointed out that the design and generation of random fields which exhibit specific properties is an attractive subject. A key property is the spatial correlation. In relation to this point, in Ref. [9] it is shown that a random diffuser under coherent illumination with a circular ring slit produces string or network like structures in the speckle patterns which was called speckle clustering. In that work, the random Koch fractal has approximately a ring-slit shape. By considering this behavior, it is concluded that the origin of the speckle clustering stems from the input object ring-like structure. It is shown that the illumination of an object with a ring slit aperture causes relatively long and ringing correlation tails in the speckle distribution. Besides, the snake-like feature of the speckles due to the ring slit aperture is attributed to the aperture circular symmetry [10]. Also, the Fraunhofer diffraction pattern formed by a plane wave incident onto a ring slit attached to a diffuser presents a cluster structure.

In Ref. [11] is analyzed the image formed by a lens having a multiple aperture pupil of an input random diffuser illuminated by a plane wave. When the pupil is a ring slit mask, the intensity pattern observed in the image plane resembles the patterns observed by Ibrahim et al. [10]. Therefore, the "cluster" terminology is maintained to mention the described experiment. In that work, it is replaced the ring slit mask by a pupil consisting of multiple apertures regularly distributed onto a ring resulting as in Ref. [9] cluster-like speckle patterns. The theoretical and experimental results suggest that the periodic arrangement of apertures produce a more directional sub-speckle distribution. It consists of tiny spots regularly distributed defined as a "regular cluster". We showed that the cluster structure origin is a consequence of the multiple speckle modulation generated by the multiple apertures.

The previous analysis only considers multiple aperture pupils distributed on a circumference. We found that by using the mentioned pupil the speckle image distributions exhibit cluster structures with a remarkable regularity that replicate the apertures distribution. This feature motivates us to develop a deeper analysis by including different closed curves. Then, we want to extend the previous analysis to a wider family of closed curves. Note that any closed curve could be discretized by using an arrangement of identical tiny apertures. The intensity distribution obtained can be explained from a multiple apertures interference point of view. In this proposal, different aperture arrangements distributed on closed curves are utilized as pupil mask (for instance: square, ellipse, etc.). It is observed that the cluster structure features depend on the aperture size and the closed curve geometry (shape and size).

We demonstrate that directional structures appear inside the volume speckle. Then, it could be possible to generate highly localized three-dimensional intensity patterns.

Optical micromanipulation requires novel light beams to operate and to trap microscopic particle [12]. This idea suggests that the cluster could be useful to tweeze several particles at once, due to their highly directional nature. Therefore, it seems to be adequate to deal with different cluster structure which can be generated by different closed curves in order to choose the condition to trapping.

2. Cluster structure analysis

In a previous work, we analyze the subjective speckle generated when the image of a random diffuser is formed by a lens with a multiple aperture pupil. In the resulting speckle pattern the intensity distribution exhibits a modulation structure defined as a "cluster". This structure is a consequence of the interference of the speckle patterns produced through each aperture. In Ref. [11] a theoretical and experimental cluster analysis formation is presented. In particular, it was demonstrated that speckle patterns obtained by using multiple aperture pupils along a circumference present an intensity distribution which resembles unit cells that duplicate themselves. These repeated structures are defined as "regular clusters". It is possible to control the regular cluster appearance by changing the pupil geometrical parameters (the number of circular apertures, the aperture radius, the circumference radius and the uniformity of the apertures distribution along the curve).

Besides, in Ref. [11] is demonstrated that the autocorrelation intensity can be expressed in terms of the output intensity when a diffuser (multiplicity of single scatters) is employed. This result is the starting point to understand the cluster generation.

Let us investigate in the mentioned framework the behavior of the resulting speckle patterns, when different closed curves utilized as geometrical loci of identical apertures are employed. Note that any slit-like closed curve can be considered as a limiting case of an arrangement of discrete identical tiny apertures.

Let us consider the set-up shown in Fig. 1. In order to analyze the proposal, we generate the subjective speckle. A coherent and linearly polarized plane wave from a Nd YAG laser (wavelength λ = 532 nm) illuminates a random diffuser (x–y plane). By using a lens L an image of the diffuser is formed at the X–Y plane. A multiple aperture pupil mask P is attached in front of the imaging lens. The focal length f of the lens L located in the u–v plane is 52 mm. The distance from the diffuser to the lens is Z_0 = 56 mm and the distance from the lens to the image plane is Z_C = 717 mm. The lens law applies, therefore, $Z_c = \frac{fz_0}{z_0 - f}$ The average speckle size

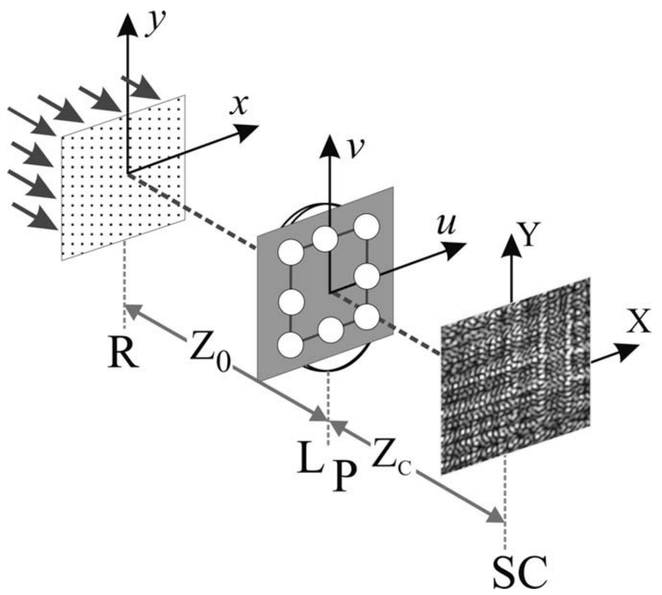

Fig. 1. Experimental set-up (R: random diffuser with rms 30 μm; L: lens; P: pupil mask; Z_0 = 56 mm: object distance; Z_C = 717 mm: image distance and SC: image speckle cluster distribution).

depends on the pupil aperture diameter D and the distance Z_C, and can be expressed by the speckle depth $\langle S_z \rangle = \lambda (\frac{z_c}{D})^2$ and the speckle width $\langle S_x \rangle = (\frac{\lambda z_c}{D})$. If the focal length f increases by maintaining the distance Z_0 fixed, then $\langle S_z \rangle$ and $\langle S_x \rangle$ increase as well. As it is mentioned, it is possible to represent any closed slit-like curve as an arrangement of N discrete tiny apertures (a_1, a_2, …, a_N) distributed along the curve. Note that in all cases to be analyzed, circular apertures are considered. By referring to Fig. 1, a speckled image of the random diffuser is produced through each aperture. Besides, the complex amplitudes of waves going through different apertures are statistically independent of each other, because different components of the angular spectrum of the scattered light are accepted by them. Moreover, the resulting speckle pattern appears as the interference of speckle distributions produced by each aperture pair because they are coherent. The average period of each fringe pattern that modulates the speckles is $\Lambda = (\frac{\lambda z_c}{d})$ where d represents the distance between the centers of each aperture pair. In summary, a complex system of fringes existing in the whole volume of each individual speckle modulates the image pattern recorded by using a CCD camera.Let us consider the necessary theoretical assumptions in order to analyze the cluster formation. We use the random walk model and a new approximation for the quadratic phase factors to simulate cluster speckle patterns structures [13]. The mentioned approach implies to consider a pupil function formed by N non-overlapping apertures. The amplitude transmission function corresponding to the hth aperture is defined as $a_h(u, v)$. In the detailed model, the speckle field in the diffuser plane is originated by m scattering points, each one with amplitude, phase and positions associated. Then, the field in the diffuser plane (see Fig. 1) can be expressed as $U_0(x, y) = \sum_{q=1}^{m} \tilde{U}_q(x, \ y) \exp(j\phi_q)$ where $\tilde{U}_q(x, y)$ is the field amplitude and ϕ_q is the phase of discrete scattering points (j is the imaginary unit). We suppose that the scattering points are uniformly, randomly distributed in the diffuser plane and have random amplitudes in the range [0; 1] and have phases randomly and uniformly distributed in the range [0; 2π] (fully developed speckle regime).

For N identical apertures arbitrary distributed, the intensity pattern at the image plane (X–Y plane) is given by:

$$
\begin{aligned}
I_i(X,Y) \propto \Bigg\{ & \sum_{q=1}^{m} U_q^2 \Big[N + 2 \sum_{h,h'=1 h>h'}^{N} \cos(\phi_{qh} - \phi_{qh'} + \phi_h - \phi_{h'})\Big]. \\
& + 2 \sum_{q,q'=1 q>q'}^{m} U_q U_{q'} \Bigg[\sum_{h=1}^{N} \cos(\phi_q - \phi_{q'} + \phi_{qh} - \phi_{q'h}) \\
& + \sum_{h,h'=1 h>h'}^{N} \big(\cos(\phi_q - \phi_{q'} + \phi_{qh} - \phi_{q'h'} + \phi_h - \phi_{h'}) \\
& + \cos(\phi_q - \phi_{q'} + \phi_{qh'} - \phi_{q'h} + \phi_{h'} - \phi_h)\big)\Bigg] \Bigg\}
\end{aligned} \tag{1}
$$

where

$$\phi_{qh} = -\frac{kr_h}{Z_0}(x_q \cos\alpha_h + y_q \sin\alpha_h) + \frac{k}{2Z_0}(x_q^2 + y_q^2),$$

$$\phi_h = -\frac{kr_h}{Z_C}(X\cos\alpha_h + Y\sin\alpha_h),$$

$$U_q(X,Y) = \int\int dxdy\tilde{U}_q(x,y)A(x,y;X,Y),$$

$$A(x,y;X,Y) = \int\int dudv\,a(u,v)exp\left\{jk\left[u\left(\frac{x}{Z_0}+\frac{X}{Z_C}\right) + v\left(\frac{y}{Z_0}+\frac{Y}{Z_C}\right)\right]\right\}$$

$U_q(X,Y)$ is the amplitude in the image plane; $A(x,y;X,Y)$ represents the response of the system to an impulsive input; $k = \frac{2\pi}{\lambda}$; $a(u,v)$ represents the amplitude transmission corresponding to an aperture; (x_q, y_q) is the random position of the qth diffuser scattering point; r_h is the distance from the geometrical centre of the hth aperture to the optical axis and α_h is the angle that forms the segment r_h with the u-axis (see Fig. 1). The first and the second terms in Eq. (1) represent the superposition of intensities due to a single scattering point of the input diffuser. The third and the fourth terms represent the contribution due to a pair of scattering points.

The theoretical simulations presented are obtained by using Matlab software package.

In Fig. 2 are displayed the computer simulated patterns belonging to pupil apertures regularly distributed on a circumference, an ellipse, a square and a triangle, respectively. Note that in all cases, each aperture pair is associated with an elementary fringe system which is characterized by: an average frequency directly proportional to the apertures separation; it runs perpendicularly to the line joining the apertures; and the average modulation depth depends on the multiplicity of the aperture pairs that contribute to this fringe system. Note that, the speckle size is not apparent in the results of Fig. 2 because the inner modulations dominate the pattern.

It should be emphasized that highly localized bright spots constitute the elementary components of the cluster structure itself. It is observed that the regular cluster exhibits a symmetry clearly associated with the symmetry of the curve where the apertures lie. When apertures are distributed with mirror-like symmetry the regular cluster resembles the geometry of the input arrangement. Moreover, even when the aperture arrangement lacks of mirror symmetry the regular cluster adopts a distribution which can be interpreted as a symmetric version of the input pupil.

By varying the ellipse eccentricity, the cluster evolves from a structure associated with a circular slit arrangement to a structure which resembles the pattern produced by a double slit aperture. This behavior is apparent by observing the theoretical simulation of Fig. 2a and 2b and Fig. 3 shows that by changing the major and minor ellipse axis, the pattern clearly follows this behavior that of course is equivalent rotating the pattern.

Fig. 4 shows simulated results corresponding to a fixed number of apertures regularly distributed along squares of different sizes. As is observed in Fig. 2, the cluster structure replicates the aperture distributions. It is apparent in Fig. 4 that if the square side L increases, then the regular cluster side decreases accordingly. Note that the cluster size is of the order of $\frac{\lambda z_C N}{p}$ where p is the closed

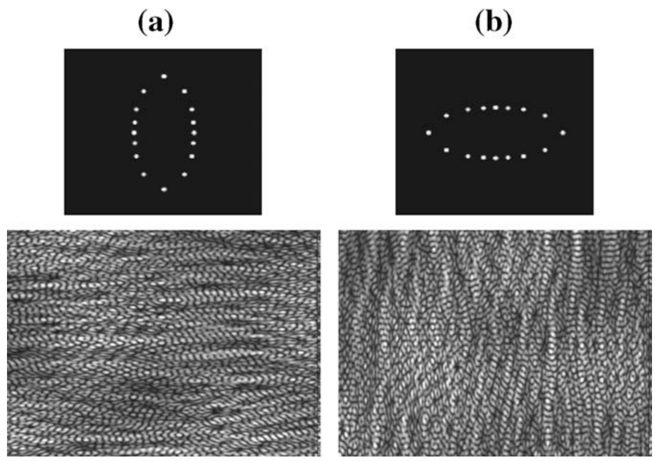

Fig. 3. Computer simulated cluster structure corresponding to an ellipse aperture radius $R = 1.5$ mm and apertures number $N = 16$: (a) $A = 45$ mm, $B = 20$ mm and (b) $A = 20$ mm, $B = 45$ mm.

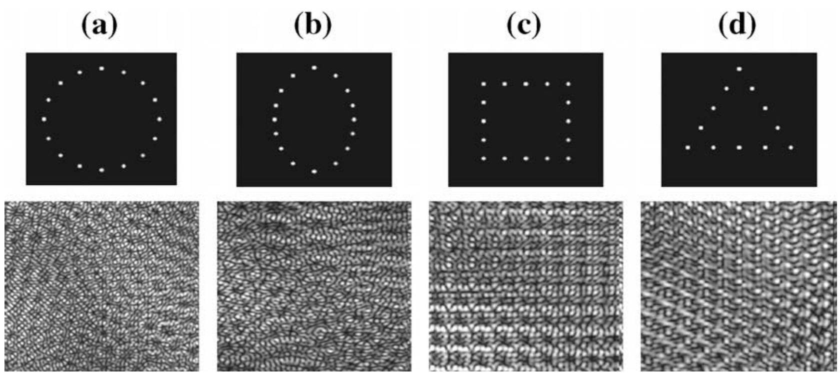

Fig. 2. Computer simulated cluster structure corresponding to different closed curves: (a) circumference with a radius $C = 45$ mm, aperture radius $R = 1.5$ mm, apertures number $N = 16$; (b) ellipse with major axis $A = 45$ mm, minor axis $B = 30$ mm, aperture radius $R = 1.5$ mm and apertures number $N = 16$; (c) square with apertures number $N = 16$ and square side/aperture radius $L/R = 60$ and (d) triangle apertures number $N = 12$ and triangle side/aperture radius $L/R = 60$.

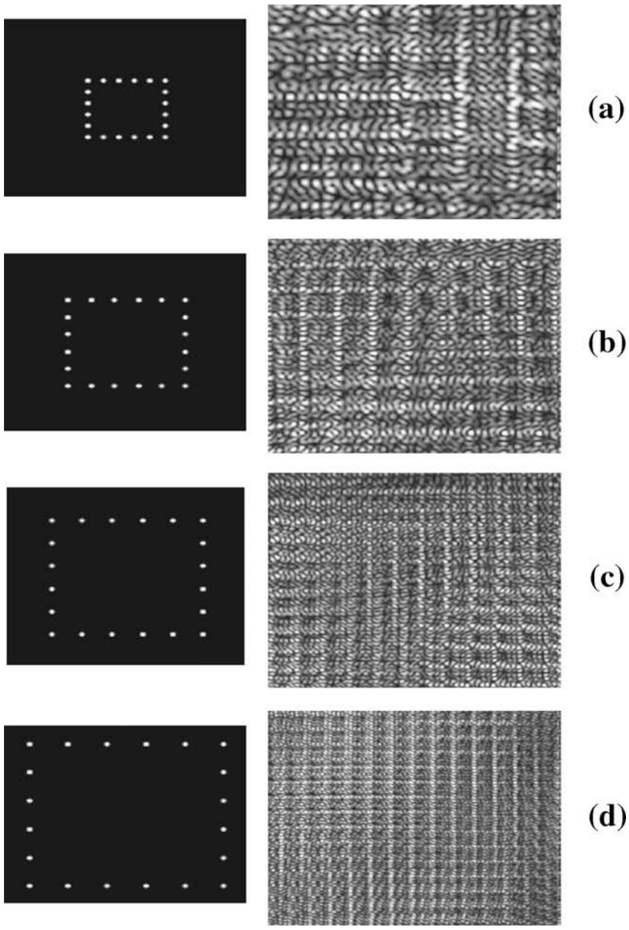

Fig. 4. Computer simulated cluster structure corresponding to a square with apertures number $N = 20$ and (a) $L/R = 40$; (b) $L/R = 60$; (c) $L/R = 80$ and (d) $L/R = 100$ (aperture radius $R = 1.5$ mm).

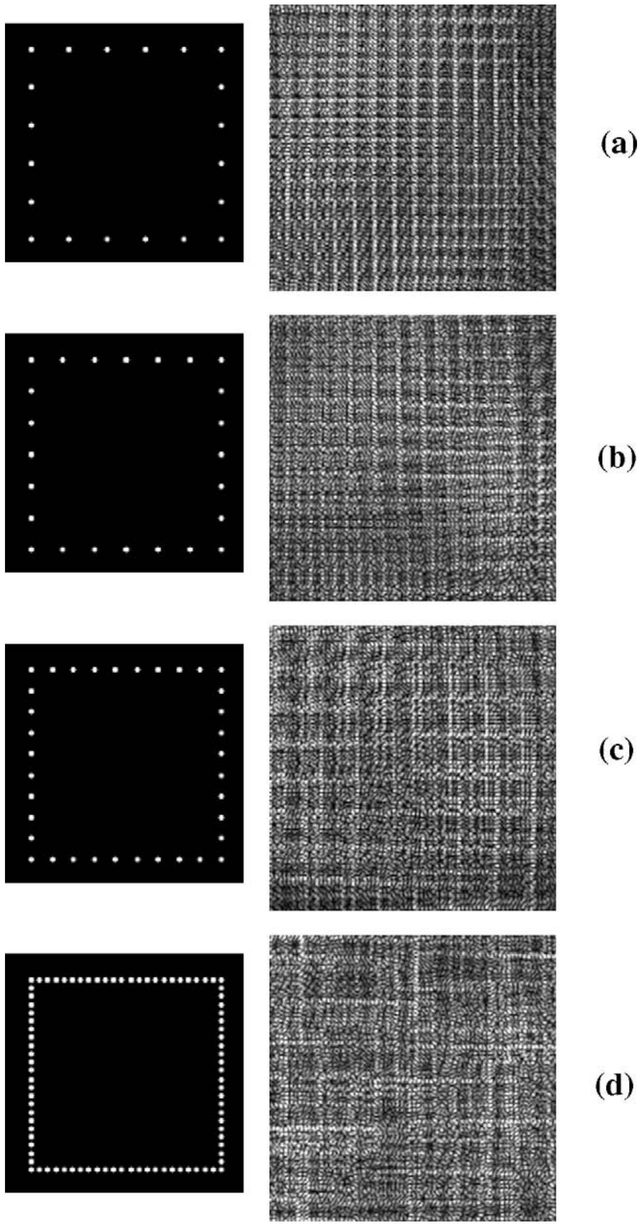

Fig. 5. Computer simulated cluster structure corresponding to a square with $L/R = 60$, aperture radius $R = 1.5$ mm and aperture number: (a) $N = 20$; (b) $N = 24$; (c) $N = 36$ and (d) $N = 96$ apertures.

curve perimeter and $p > 2RN$ where N is the number of apertures and R is the aperture radius.

Fig. 5 displays simulated results corresponding to identical squares with different number of apertures. As the number of apertures increases the number of bright tiny spot increases in the same proportion and the regular cluster radius becomes longer. However, if this number increases further, it becomes difficult to observe the regular arrangement. In fact, more apertures imply higher contribution in the number of speckles that interfere to generate the pattern. Therefore, it is apparent that a case exists where the regular cluster has "better visibility" (see Fig. 5).

The necessary geometrical conditions which lead up to the regular cluster appearance could be explained as follows. When passing from one to several apertures in the pupil mask, the autocorrelation function gradually begins to form the long ringing lobes [10,11]. It implies that each image point gradually receives contributions from more and more speckles, showing the statistical nature of the phenomenon that breaks the regular cluster structure.

Note that in the synthesis of the cluster structure, the addition of apertures on the closed curve implies generation of new modulated fringe systems which turn out to be the predominant fringe systems to build up the cluster. It is supposed that the apertures are distributed uniformly along the closed curve.

It is interesting to emphasize a common feature observed in the regular clusters analyzed: their sizes increase in two situations. (a) When the number of apertures is increased, keeping fixed the curve size. (b) When the curve size diminishes keeping fixed the number of apertures. These two situations suggest that the size of the regular cluster is proportional to the "density" of apertures per unit curve length.

The behavior observed and discussed in the simulated results displayed in Figs. 2–5 is experimentally confirmed. Fig. 6 shows that if the square side is maintained the regular cluster increases its size in accordance with the aperture number increasing. Fig. 7 corresponds to maintain the number of apertures regularly distributed but changing the squares sizes. Fig. 7 confirms that if the square size increases, then the cluster unit cell decreases. The experimental results of Figs. 6 and 7 must be compared with the simulations of Figs. 4 and 5, respectively.

Fig. 8 shows experimental results corresponding to apertures distributed forming different slit-like curves (square, circumference and triangle). The patterns are in good agreement with those of Fig. 2.

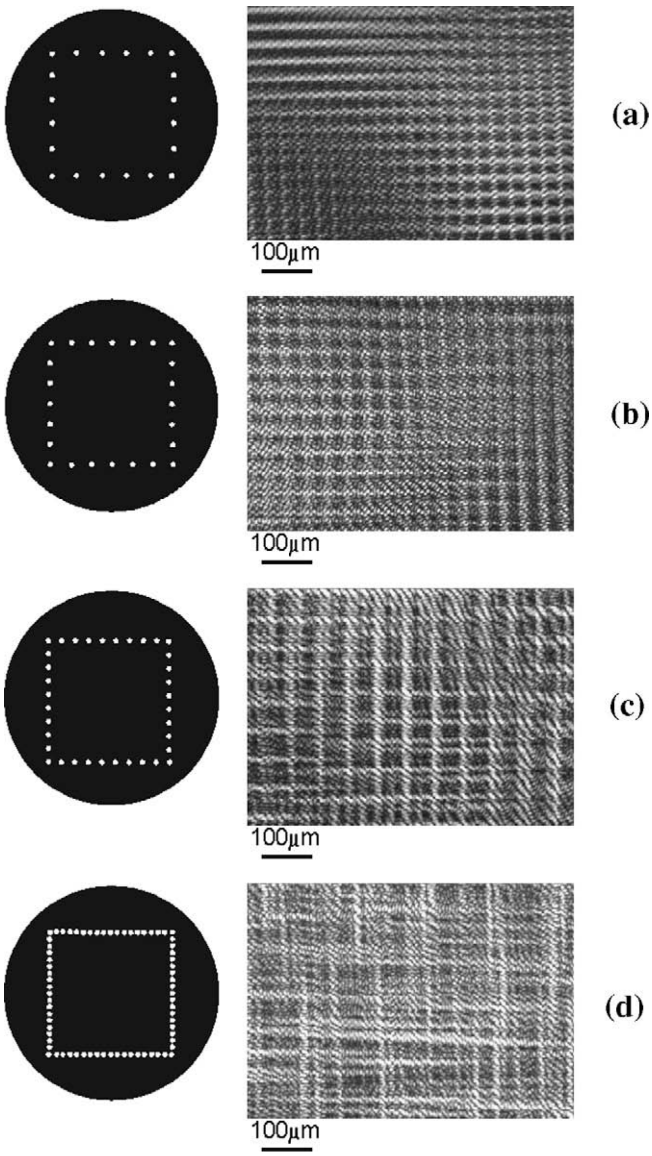

Fig. 6. Experimental results corresponding to apertures distributed on 25 mm square side and $R = 0.4$ mm and with aperture number: (a) $N = 20$; (b) $N = 24$; (c) $N = 36$ and (d) $N = 92$.

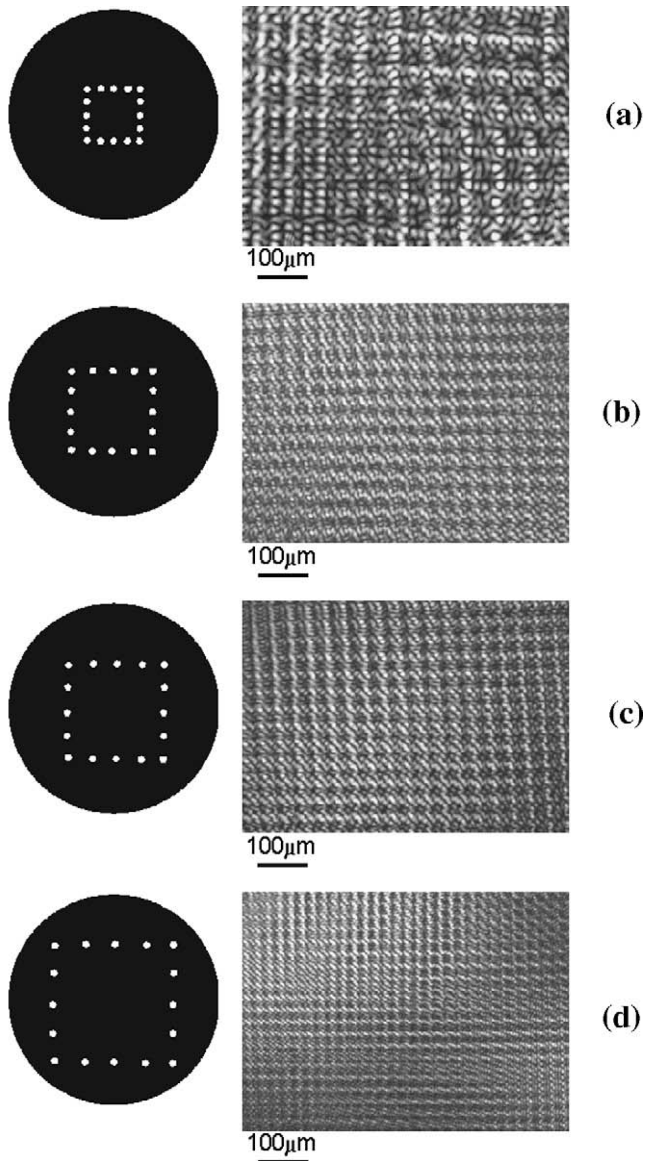

Fig. 7. Experimental results corresponding to 16 apertures with radius $R = 0.4$ mm distributed on a square side: (a) $L = 25$ mm; (b) $L = 20$ mm; (c) $L = 15$ mm and (d) 10 mm.

It is valid that a decreasing aperture radius implies that the tiny bright spots that form the regular cluster are better defined. Besides, those spots that are located outside the regular cluster tend to diminish producing a "cleaner" pattern.

It should be emphasized that the behavior observed in the square curve is also observed when the apertures are periodically distributed in any other closed curves, for instance triangle, ellipse, etc.

As final remarks some features of the regular cluster can be pointed out:

(1) It exhibits a regular distribution of spots that resembles the apertures arrangement in the pupil.
(2) The number of spots that form the regular cluster is proportional to the apertures number.
(3) The size of the regular cluster changes in accordance with the closed curve characteristic parameters (radius, side, eccentricity, etc.) where the apertures are distributed.
(4) There is an inverse relation between the closed curve characteristic parameter and the size of the cluster that replicates in the pattern.
(5) The cluster structure does not depend on the aperture radius.

3. Conclusions

When a coherently illuminated diffuser is imaged by a lens having a multiple apertures mask a cluster-like speckle pattern appears showing that apertures distribute along a closed curve (ring-like, square-like and triangle-like pupil).

The results allow concluding that: there exists a limited range in the apertures number where the regular cluster is observed and a determined number of apertures for which the regular cluster is better defined. Indeed, when the number of apertures is as high as to fade out the regular cluster structure, it can be retrieved by appropriately diminishing the aperture size. On the contrary, when a continuous slit pupil is used the cluster structure randomizes its appearance.

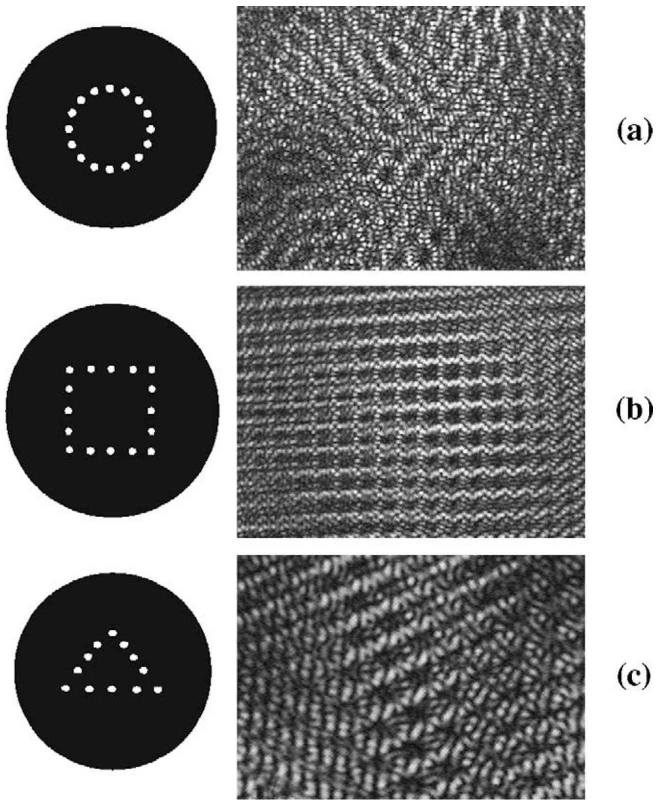

Fig. 8. Experimental results corresponding to apertures with radius $R = 0.4$ mm distributed on a: (a) circumference (circumference radio $C = 30$ mm, $N = 16$); (b) square ($L = 25$ mm, $N = 16$) and (c) triangle ($L = 20$ mm, $N = 12$).

Note that when apertures are distributed with mirror symmetry the regular cluster resembles the geometry of the input arrangement. Nevertheless, despite the lack of mirror symmetry of the aperture arrangement, the regular cluster forms itself as a symmetric version of the input pupil.

Finally, it can be inferred that the regular cluster size is governed by the apertures number and the curve size where the apertures are distributed. These features could be combined to define an aperture density per unit length in order to assess the cluster behavior.

These results give the tools to manipulate the three-dimensional intensity distributions inside speckle patterns which can be exploited in application related with optical tweezers. In speckle like-cluster, light volumes are reduced several times compared to usual speckles. Then, multiple and random trap volumes could be created with this technique. Therefore, it seems to be convenient to deal with different cluster structure which can be generated by different closed curves in order to choose the adequate condition to trapping.

Acknowledgments

This research was performed under the grants: CONICET PIP No. 112-200801-00863 (Argentina), ANCYT PICT 1167 (Argentina), Facultad de Ingeniería, Universidad Nacional de La Plata No. 11/I125 (Argentina).

References

[1] M. Francon, Laser Speckle and Applications in Optics, Academic Press, New York, 1979.
[2] J.C. Dainty (Ed.), Laser Speckle and Related Phenomena, Springer-Verlag, Berlin, 1975.
[3] M. Tebaldi, L. Angel Toro, M. Trivi, N. Bolognini, Opt. Commun. 182 (2000) 95.
[4] M. Tebaldi, L. Angel Toro, M. Trivi, N. Bolognini, Opt. Eng. 39 (2000) 3232.
[5] L. Angel Toro, M. Tebaldi, N. Bolognini, M. Trivi, J. Opt. Soc. Am. A 17 (2000) 107.
[6] L. Angel, M. Tebaldi, M. Trivi, N. Bolognini, Opt. Lett. 27 (2002) 506.
[7] L. Angel, M. Tebaldi, N. Bolognini, Appl. Opt. 46 (2007) 2676.
[8] L. Angel, M. Tebaldi, N. Bolognini, Opt. Commun. 274 (2007) 23.
[9] K. Uno, J. Uozumi, T. Asakura, Opt. Commun. 114 (1995) 203.
[10] M. Ibrahim, J. Uozumi, T. Asakura, Optik 106 (1997) 33.
[11] A. Lencina, M. Tebaldi, P. Vaveliuk, N. Bolognini, Wave. Random Complex Media 17 (2007) 29.
[12] P. Prentice, M. MacDonald, T. Frank, A. Cuschier, G. Spalding, W. Sibbett, P. Campbell, K. Dholakia, Opt. Exp. 12 (2004) 593.
[13] A. Lencina, P. Vaveliuk, M. Tebaldi, N. Bolognini, Opt. Lett. 28 (2003) 1748.